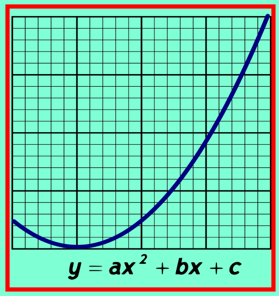
Math 125

Fall 2021

Lecture 37



Class QZ 37

1) Divide: $\dfrac{1-i}{1+i} = \dfrac{(1-i)(1-i)}{(1+i)(1-i)} = \dfrac{\cancel{1} - i - i + \cancel{i^2}}{1 \cancel{-i} \cancel{+i} - i^2} = \dfrac{-\cancel{2}i}{\cancel{2}} = \boxed{-i}$ ✓

even

2) Simplify: $i^{400} = (i^2)^{200} = (-1)^{200} = \boxed{1}$ ✓

odd

3) Simplify: $i^{11} = i^{10} \cdot i = (i^2)^5 \cdot i = (-1)^5 \cdot i = \boxed{-i}$ ✓

Simplify

1) $5\sqrt{-8} + 3\sqrt{-18}$

$= 5\underbrace{\sqrt{4}}_{2}\sqrt{2}\sqrt{-1} + 3\underbrace{\sqrt{9}}_{3}\sqrt{2}\sqrt{-1} = 5\cdot 2\sqrt{2}\,i + 3\cdot 3\sqrt{2}\,i$

$= 10\sqrt{2}i + 9\sqrt{2}i$

$= \boxed{19\sqrt{2}i}$

$= \boxed{19i\sqrt{2}}$

2) $(5-3i)^2 = (5-3i)(5-3i)$

$= 25 - 15i - 15i + 9i^2$

$= 25 - 30i + 9(-1)$

$= 25 - 30i - 9$

$= \boxed{16 - 30i}$

Re. Part = 16

Im. " = −30

3) $\frac{5-i}{3-2i}$

$= \frac{(5-i)(3+2i)}{(3-2i)(3+2i)} = \frac{15 + 10i - 3i - 2i^2}{9 + \cancel{6i} - \cancel{6i} - 4i^2} = \frac{15 + 7i - 2(-1)}{9 - 4(-1)}$

$= \frac{17 + 7i}{13} = \boxed{\frac{17}{13} + \frac{7}{13}i}$

Rationalize the denominator:

1) $\frac{6}{\sqrt{3}}$

$= \frac{6\cdot\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{9}}$

$= \frac{\overset{2}{\cancel{6}}\sqrt{3}}{\cancel{3}_1} = \frac{2\sqrt{3}}{1} = \boxed{2\sqrt{3}}$

2) $\frac{\sqrt{15}}{\sqrt{5}-\sqrt{3}} \cdot \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})}$

$= \frac{\sqrt{15}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$

$= \frac{\sqrt{75} + \sqrt{45}}{\sqrt{25} + \cancel{\sqrt{15}} - \cancel{\sqrt{15}} - \sqrt{9}}$

$= \frac{\sqrt{25}\sqrt{3} + \sqrt{9}\sqrt{5}}{5 - 3} = \boxed{\frac{5\sqrt{3} + 3\sqrt{5}}{2}}$

Solve & check:

$\sqrt{2x+1}+7=x$

Isolate the radical

$\sqrt{2x+1}=x-7$

Index = 2

$(\sqrt{2x+1})^2=(x-7)^2$

$2x+1=(x-7)(x-7)$

$2x+1=x^2-7x-7x+49$

$2x+1=x^2-14x+49$

$x^2-14x+49-2x-1=0$

$x^2-16x+48=0$

$(x-12)(x-4)=0$

$x-12=0 \qquad x-4=0$

$x=12 \qquad \cancel{x=4}$ E.S.

$\{12\}$

$\sqrt{2x+1}+7=x$

Check $\boxed{x=12}$ ✓

$\sqrt{2(12)+1}+7=12$

$\sqrt{25}+7=12$

$5+7=12$

$12=12$

True

Check $\cancel{x=4}$

$\sqrt{2(4)+1}+7=4$

$\sqrt{9}+7=4$

$3+7=4$

$10=4$

False

Solve & check

$\sqrt{x-8}-\sqrt{x}=-2$

Isolate one radical

Index = 2

$\sqrt{x-8}=\sqrt{x}-2$

$(\sqrt{x-8})^2=(\sqrt{x}-2)^2$

$x-8=(\sqrt{x}-2)(\sqrt{x}-2)$

$x-8=(\sqrt{x})^2-2\sqrt{x}-2\sqrt{x}+4$

$x-8=x-4\sqrt{x}+4$

Isolate the radical

$4\sqrt{x}=\cancel{x}+4-\cancel{x}+8$

$4\sqrt{x}=12$

Divide by 4

$\sqrt{x}=3$

$(\sqrt{x})^2=(3)^2$

$\boxed{x=9}$ ✓

$\{9\}$

Check

$\sqrt{x-8}-\sqrt{x}=-2$

$\sqrt{9-8}-\sqrt{9}=-2$

$\sqrt{1}-3=-2$

$1-3=-2$ ✓

Your turn:

$\sqrt{x-7} + \sqrt{x} = 7$

$\sqrt{x-7} = 7 - \sqrt{x}$

$(\sqrt{x-7})^2 = (7 - \sqrt{x})^2$

$x - 7 = (7 - \sqrt{x})(7 - \sqrt{x})$

$x - 7 = 49 - 7\sqrt{x} - 7\sqrt{x} + (\sqrt{x})^2$

$x - 7 = 49 - 14\sqrt{x} + x$

$\cancel{x} - 7 - 49 - \cancel{x} = -14\sqrt{x}$

$-56 = -14\sqrt{x}$

Divide by -14

$4 = \sqrt{x}$ $\quad x = 16$

$4^2 = (\sqrt{x})^2$ $\quad x = 16$

$\{16\}$

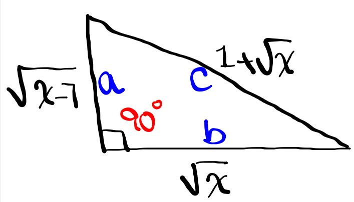
Check

$\sqrt{x-7} + \sqrt{x} = 7$

$\sqrt{16-7} + \sqrt{16} = 7$

$3 + 4 = 7 \checkmark$

Find all 3 sides of the triangle below:



Right-Triangle

Pythagorean Thrm

$a^2 + b^2 = c^2$

$(\sqrt{x-7})^2 + (\sqrt{x})^2 = (1 + \sqrt{x})^2$

$x - 7 + x = (1 + \sqrt{x})(1 + \sqrt{x})$

$2x - 7 = 1 + \sqrt{x} + \sqrt{x} + (\sqrt{x})^2$

$2x - 7 = 1 + 2\sqrt{x} + x$

$2x - 7 - 1 - x = 2\sqrt{x}$

$x - 8 = 2\sqrt{x}$

$x - 8 = 2\sqrt{x}$

Square both sides

$(x-8)^2 = (2\sqrt{x})^2$

$(x-8)(x-8) = 4x$

$x^2 - 8x - 8x + 64 = 4x$

$x^2 - 16x + 64 = 4x$

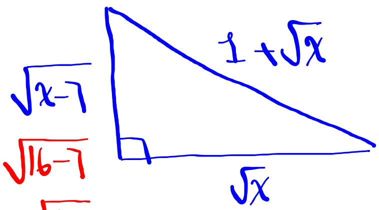
$x^2 - 16x + 64 - 4x = 0$

$x^2 - 20x + 64 = 0$

$(x-16)(x-4) = 0$

$x - 16 = 0 \qquad x - 4 = 0$

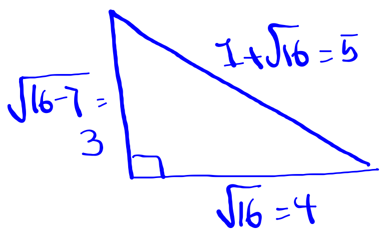
$x = 16 \qquad \cancel{x = 4}$



$\sqrt{16-7}$

$\sqrt{9}$

$= 3$



Three sides are 3, 4, and 5 of same units.

Solve and check:

$\sqrt{x-4} + \sqrt{x+1} = 5$

$\sqrt{x-4} = 5 - \sqrt{x+1}$

$(\sqrt{x-4})^2 = (5 - \sqrt{x+1})^2$

$x - 4 = (5 - \sqrt{x+1})(5 - \sqrt{x+1})$

$x - 4 = 25 - 5\sqrt{x+1} - 5\sqrt{x+1} + (\sqrt{x+1})^2$

$x - 4 = 25 - 10\sqrt{x+1} + x + 1$

$\cancel{x} - 4 - 25 - \cancel{x} - 1 = -10\sqrt{x+1}$

$-30 = -10\sqrt{x+1}$

Divide by -10

$3 = \sqrt{x+1} \qquad 3^2 = (\sqrt{x+1})^2$

$x = 8 \longleftarrow 9 = x + 1$

$\{8\}$

Check $x = 8$

$\sqrt{x-4} + \sqrt{x+1} = 5$

$\underbrace{\sqrt{8-4}} + \underbrace{\sqrt{8+1}} = 5$

$2 + 3 = 5 \checkmark$

Rationalize the denominator:

1) $\frac{4}{\sqrt{6}}$

$= \frac{4 \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{4\sqrt{6}}{\sqrt{36}}$

$= \frac{\cancel{4}^{2}\sqrt{6}}{\cancel{6}_{3}} = \boxed{\frac{2\sqrt{6}}{3}}$

2) $\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}} \cdot \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

$= \frac{\sqrt{49}+\sqrt{21}+\sqrt{21}+\sqrt{9}}{\underbrace{\sqrt{49}}_{7} \cancel{+\sqrt{21}} \cancel{-\sqrt{21}} \underbrace{-\sqrt{9}}_{3}}$

$= \frac{7+2\sqrt{21}+3}{7-3} = \frac{10+2\sqrt{21}}{4}$

$= \frac{\cancel{2}(5+\sqrt{21})}{\cancel{4}_{2}}$

$= \boxed{\frac{5+\sqrt{21}}{2}}$

Rationalize the numerator:

$\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}} \cdot \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{\sqrt{49} \cancel{-\sqrt{21}} \cancel{+\sqrt{21}} -\sqrt{9}}{\sqrt{49} -\sqrt{21} -\sqrt{21} +\sqrt{9}}$

$= \frac{7 - 3}{7 - 2\sqrt{21} + 3} = \frac{4}{10-2\sqrt{21}}$

$= \frac{\cancel{4}^{2}}{\cancel{2}(5-\sqrt{21})} = \frac{2}{5-\sqrt{21}}$

$\frac{4/2}{10/2 - 2\sqrt{21}/2}$

$= \frac{2}{5-\sqrt{21}}$

$f(x) = \sqrt{x+5}$

Square-root Function

$f(-4) = \sqrt{-4+5}$
$= \sqrt{1}$
$= \boxed{1}$

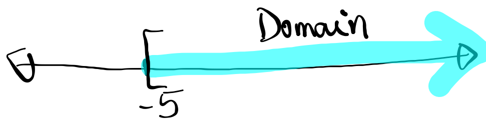
$f(4) = \sqrt{4+5}$
$= \sqrt{9}$
$= \boxed{3}$

Domain of $f(x)$

Square-Root → even root → Radicand ≥ 0

$x+5 \geq 0$

$x \geq -5$



Interval notation $[-5, \infty)$